SOLUTIONS TO PROBLEMS

10.1 (i) Disagree. Most time series processes are correlated over time, and many of them strongly correlated. This means they cannot be independent across observations, which simply represent different time periods. Even series that do appear to be roughly uncorrelated – such as stock returns – do not appear to be independently distributed, as you will see in Chapter 12 under dynamic forms of heteroskedasticity.

(ii) Agree. This follows immediately from Theorem 10.1. In particular, we do not need the homoskedasticity and no serial correlation assumptions.

(iii) Disagree. Trending variables are used all the time as dependent variables in a regression model. We do need to be careful in interpreting the results because we may simply find a spurious association between y_t and trending explanatory variables. Including a trend in the regression is a good idea with trending dependent or independent variables. As discussed in Section 10.5, the usual R-squared can be misleading when the dependent variable is trending.

(iv) Agree. With annual data, each time period represents a year and is not associated with any season.

10.2 We follow the hint and write

$$gGDP_{t-1} = \alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + u_{t-1},$$

and plug this into the right-hand-side of the int_t equation:

$$\begin{aligned} int_t &= \gamma_0 + \gamma_1(\alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + u_{t-1} - 3) + v_t \\ &= (\gamma_0 + \gamma_1\alpha_0 - 3\gamma_1) + \gamma_1\delta_0 int_{t-1} + \gamma_1\delta_1 int_{t-2} + \gamma_1 u_{t-1} + v_t. \end{aligned}$$

Now by assumption, u_{t-1} has zero mean and is uncorrelated with all right-hand-side variables in the previous equation, except itself of course. So

$$\text{Cov}(int,u_{t-1}) = \text{E}(int_t \cdot u_{t-1}) = \gamma_1 \text{E}(u_{t-1}^2) > 0$$

because $\gamma_1 > 0$. If $\sigma_u^2 = \text{E}(u_t^2)$ for all t then $\text{Cov}(int,u_{t-1}) = \gamma_1\sigma_u^2$. This violates the strict exogeneity assumption, TS.2. While u_t is uncorrelated with int_t, int_{t-1}, and so on, u_t is correlated with int_{t+1}.

10.3 Write

$$y^* = \alpha_0 + (\delta_0 + \delta_1 + \delta_2)z^* = \alpha_0 + LRP \cdot z^*,$$

and take the change: $\Delta y^* = LRP \cdot \Delta z^*$.

SOLUTIONS TO PROBLEMS

11.1 Because of covariance stationarity, $\gamma_0 = \text{Var}(x_t)$ does not depend on t, so $\text{sd}(x_{t+h}) = \sqrt{\gamma_0}$ for any $h \geq 0$. By definition, $\text{Corr}(x_t,x_{t+h}) = \text{Cov}(x_t,x_{t+h})/[\text{sd}(x_t)\cdot\text{sd}(x_{t+h})] = \gamma_h/(\sqrt{\gamma_0}\cdot\sqrt{\gamma_0}) = \gamma_h/\gamma_0$.

11.2 (i) $\text{E}(x_t) = \text{E}(e_t) - (1/2)\text{E}(e_{t-1}) + (1/2)\text{E}(e_{t-2}) = 0$ for $t = 1,2, \ldots$ Also, because the e_t are independent, they are uncorrelated and so $\text{Var}(x_t) = \text{Var}(e_t) + (1/4)\text{Var}(e_{t-1}) + (1/4)\text{Var}(e_{t-2}) = 1 + (1/4) + (1/4) = 3/2$ because $\text{Var}(e_t) = 1$ for all t.

(ii) Because x_t has zero mean, $\text{Cov}(x_t,x_{t+1}) = \text{E}(x_tx_{t+1}) = \text{E}[(e_t - (1/2)e_{t-1} + (1/2)e_{t-2})(e_{t+1} - (1/2)e_t + (1/2)e_{t-1})] = \text{E}(e_te_{t+1}) - (1/2)\text{E}(e_t^2) + (1/2)\text{E}(e_te_{t-1}) - (1/2)\text{E}(e_{t-1}e_{t+1}) + (1/4(\text{E}(e_{t-1}e_t) - (1/4)\text{E}(e_{t-1}^2) + (1/2)\text{E}(e_{t-2}e_{t+1}) - (1/4)\text{E}(e_{t-2}e_t) +(1/4)\text{E}(e_{t-2}e_{t-1}) = -(1/2)\text{E}(e_t^2) - (1/4)\text{E}(e_{t-1}^2) = -(1/2) - (1/4) = -3/4$; the third to last equality follows because the e_t are pairwise uncorrelated and $\text{E}(e_t^2) = 1$ for all t. Using Problem 11.1 and the variance calculation from part (i), $\text{Corr}(x_tx_{t+1}) = -(3/4)/(3/2) = -1/2$.

Computing $\text{Cov}(x_t,x_{t+2})$ is even easier, because only one of the nine terms has expectation not equal to zero: $(1/2)\text{E}(e_t^2) = ½$. Therefore, $\text{Corr}(x_t,x_{t+2}) = (1/2)/(3/2) = 1/3$.

(iii) $\text{Corr}(x_t,x_{t+h}) = 0$ for $h > 2$ because for $h > 2$, x_{t+h} depends at most on e_{t+j} for $j > 0$, while x_t depends on e_{t+j}, $j \leq 0$.

(iv) Yes, because terms more than two periods apart are actually uncorrelated, and so it is obvious that $\text{Corr}(x_t,x_{t+h}) \to 0$ as $h \to \infty$.

11.3 (i) $\text{E}(y_t) = \text{E}(z + e_t) = \text{E}(z) + \text{E}(e_t) = 0$. $\text{Var}(y_t) = \text{Var}(z + e_t) = \text{Var}(z) + \text{Var}(e_t) + 2\text{Cov}(z,e_t) = \sigma_z^2 + \sigma_e^2 + 2\cdot 0 = \sigma_z^2 + \sigma_e^2$. Neither of these depends on t.

(ii) We assume $h > 0$; when $h = 0$ we obtain $\text{Var}(y_t)$. Then $\text{Cov}(y_t,y_{t+h}) = \text{E}(y_ty_{t+h}) = \text{E}[(z + e_t)(z + e_{t+h})] = \text{E}(z^2) + \text{E}(ze_{t+h}) + \text{E}(e_tz) + \text{E}(e_te_{t+h}) = \text{E}(z^2) = \sigma_z^2$ because $\{e_t\}$ is an uncorrelated sequence (it is an independent sequence and z is uncorrelated with e_t for all t. From part (i) we know that $\text{E}(y_t)$ and $\text{Var}(y_t)$ do not depend on t and we have shown that $\text{Cov}(y_t,y_{t+h})$ depends on neither t nor h. Therefore, $\{y_t\}$ is covariance stationary.

(iii) From Problem 11.1 and parts (i) and (ii), $\text{Corr}(y_t,y_{t+h}) = \text{Cov}(y_t,y_{t+h})/\text{Var}(y_t) = \sigma_z^2/(\sigma_z^2 + \sigma_e^2) > 0$.

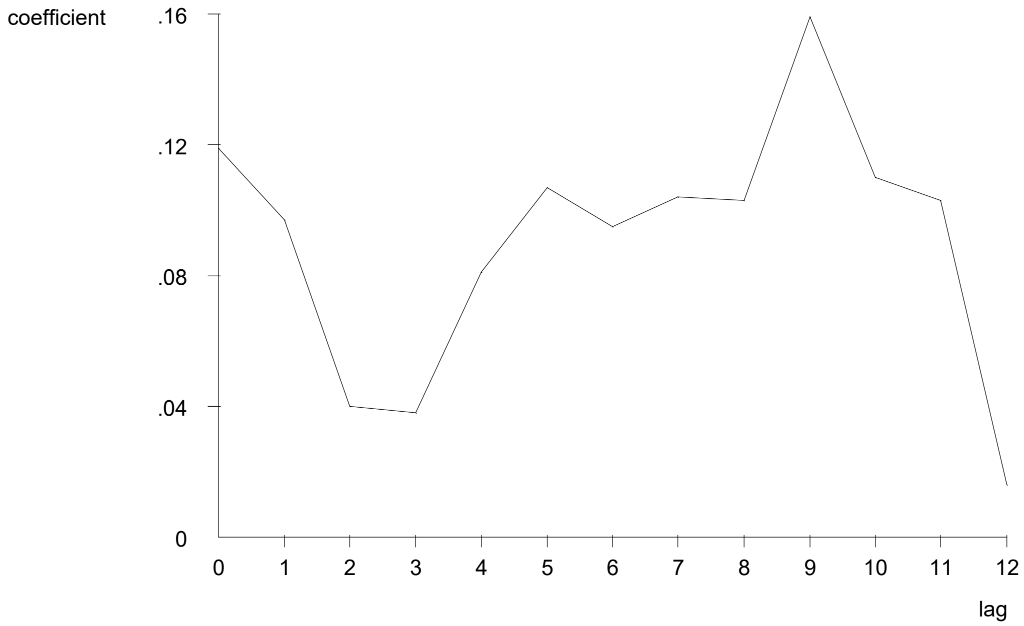
(iv) No. In fact, the correlation between y_t and y_{t+h} is the same positive value obtained in part (iii) for any $h > 0$. In other words, no matter how far apart y_t and y_{t+h} are, their correlation is always the same. Of course this is due to the presence of the time-constant variable, z.

11.4 Assuming $y_0 = 0$ is a special case of assuming y_0 nonrandom, and so we can obtain the variances from (11.21): $\text{Var}(y_t) = \sigma_e^2 t$ and $\text{Var}(y_{t+h}) = \sigma_e^2(t + h)$, $h > 0$. Because $\text{E}(y_t) = 0$ for all t (since $\text{E}(y_0) = 0$), $\text{Cov}(y_t,y_{t+h}) = \text{E}(y_t y_{t+h})$ and, for $h > 0$,

$$\begin{aligned} \text{E}(y_t y_{t+h}) &= \text{E}[(e_t + e_{t-1} + \ldots\ e_1)(e_{t+h} + e_{t+h-1} + \ldots + e_1)] \\ &= \text{E}(e_t^2) + \text{E}(e_{t-1}^2) + \ldots + \text{E}(e_1^2) = \sigma_e^2 t, \end{aligned}$$

where we have used the fact that $\{e_t\}$ is a pairwise uncorrelated sequence. Therefore, $\text{Corr}(y_t,y_{t+h}) = \text{Cov}(y_t,y_{t+h})/\sqrt{\text{Var}(y_t)\cdot\text{Var}(y_{t+h})} = t/\sqrt{t(t+h)} = \sqrt{t/(t+h}$.

11.5 (i) The following graph gives the estimated lag distribution:



By some margin, the largest effect is at the ninth lag, which says that a temporary increase in wage inflation has its largest effect on price inflation nine months later. The smallest effect is at the twelfth lag, which hopefully indicates (but does not guarantee) that we have accounted for enough lags of *gwage* in the FLD model.

(ii) Lags two, three, and twelve have t statistics less than two. The other lags are statistically significant at the 5% level against a two-sided alternative. (Assuming either that the CLM assumptions hold for exact tests or Assumptions TS.1′ through TS.5′ hold for asymptotic tests.)

(iii) The estimated LRP is just the sum of the lag coefficients from zero through twelve: 1.172. While this is greater than one, it is not much greater, and the difference could certainly be due to sampling error.

(iv) The model underlying and the estimated equation can be written with intercept α_0 and lag coefficients $\delta_0, \delta_1, \ldots, \delta_{12}$. Denote the LRP by $\theta_0 = \delta_0 + \delta_1 + \ldots + \delta_{12}$. Now, we can write $\delta_0 = \theta_0 - \delta_1 - \delta_2 - \ldots - \delta_{12}$. If we plug this into the FDL model we obtain (with $y_t = gprice_t$ and $z_t = gwage_t$)

$$\begin{aligned} y_t &= \alpha_0 + (\theta_0 - \delta_1 - \delta_2 - \ldots - \delta_{12})z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + \ldots + \delta_{12} z_{t-12} + u_t \\ &= \alpha_0 + \theta_0 z_t + \delta_1(z_{t-1} - z_t) + \delta_2(z_{t-2} - z_t) + \ldots + \delta_{12}(z_{t-12} - z_t) + u_t. \end{aligned}$$

Therefore, we regress y_t on z_t, $(z_{t-1} - z_t)$, $(z_{t-2} - z_t)$, …, $(z_{t-12} - z_t)$ and obtain the coefficient and standard error on z_t as the estimated LRP and its standard error.

(v) We would add lags 13 through 18 of $gwage_t$ to the equation, which leaves 273 – 6 = 267 observations. Now, we are estimating 20 parameters, so the df in the unrestricted model is df_{ur} = 267. Let R^2_{ur} be the R-squared from this regression. To obtain the restricted R-squared, R^2_r, we need to reestimate the model reported in the problem but with the same 267 observations used to estimate the unrestricted model. Then $F = [(R^2_{ur} - R^2_r)/(1 - R^2_{ur})](247/6)$. We would find the critical value from the $F_{6,247}$ distribution.

[Instructor's Note: As a computer exercise, you might have the students test whether all 13 lag coefficients in the population model are equal. The restricted regression is *gprice* on ($gwage$ + $gwage_{-1}$ + $gwage_{-2}$ + … $gwage_{-12}$), and the R-squared form of the F test, with 12 and 259 df, can be used.]

11.6 (i) The t statistic for H_0: $\beta_1 = 1$ is $t = (1.104 - 1)/.039 \approx 2.67$. Although we must rely on asymptotic results, we might as well use df = 120 in Table G.2. So the 1% critical value against a two-sided alternative is about 2.62, and so we reject H_0: $\beta_1 = 1$ against H_1: $\beta_1 \neq 1$ at the 1% level. It is hard to know whether the estimate is practically different from one without comparing investment strategies based on the theory ($\beta_1 = 1$) and the estimate ($\hat{\beta}_1 = 1.104$). But the estimate is 10% higher than the theoretical value.

(ii) The t statistic for the null in part (i) is now $(1.053 - 1)/.039 \approx 1.36$, so H_0: $\beta_1 = 1$ is no longer rejected against a two-sided alternative unless we are using more than a 10% significance level. But the lagged spread is very significant (contrary to what the expectations hypothesis predicts): $t = .480/.109 \approx 4.40$. Based on the estimated equation, when the lagged spread is positive, the predicted holding yield on six-month T-bills is above the yield on three-month T-bills (even if we impose $\beta_1 = 1$), and so we should invest in six-month T-bills.

(iii) This suggests unit root behavior for $\{hy3_t\}$, which generally invalidates the usual t-testing procedure.